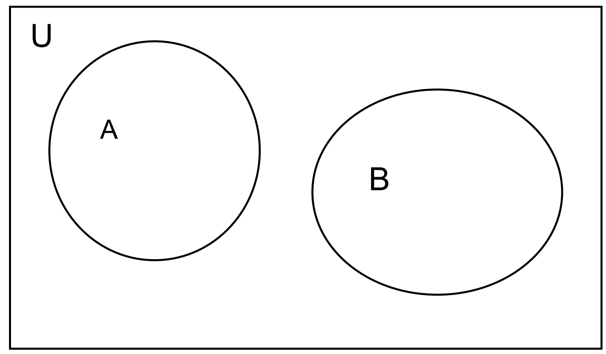
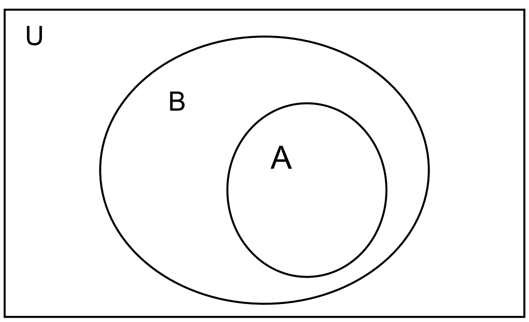
VENN DIAGRAMS AND SET OPERATIONS

Let **U** = the universal set (the set of all possible elements).
Two sets may be represented in a Venn Diagram in any of four different ways.



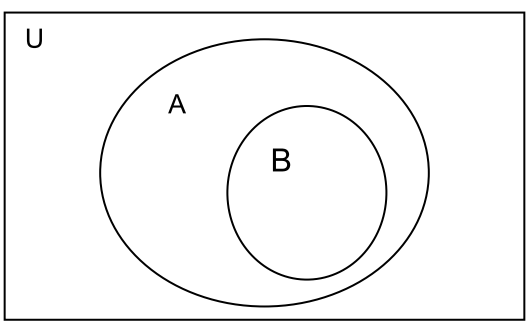
Case 1: Disjoint Sets

Two sets A and B are disjoint when they have no elements in common.



Case 2: Subsets

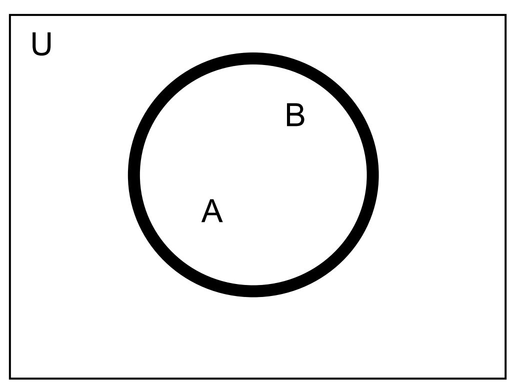
When A $\subseteq$ B, every element of set A is also an element of set B. Thus, there can be no elements in set A that are not in set B.



When B $\subseteq$ A, every element of set B is also an element of set A. Thus, there can be no elements in set B that are not in set A.

Case 3: Equal Sets

When set A = set B, all the elements of set A are elements of set B, and all the elements of set B are elements of set A.



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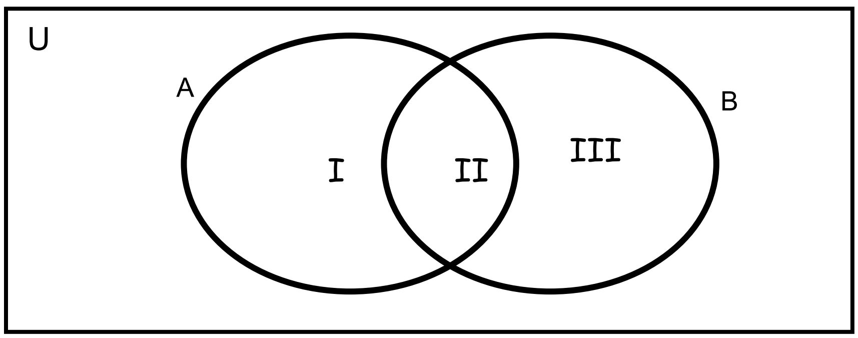
Case 4: Overlapping Sets

When sets A and B have elements in common, those elements belong to region II. The elements that belong to set A but do not belong to set B are in region I. The elements that belong to set B but do not belong to set A are in region III.



Venn Diagrams will be helpful in understanding set operations.

Compliment

The compliment of set A, symbolized by A′, is the set of all the elements in the universal set that are not in set A.

Example: Given U = { M, A, T, H, 1, 2, 0 } and A = { M, A, T, H }

Then A′ = { 1, 2, 0 }

Intersection

The intersection of sets A and B, symbolized by $A \cap B$, is the set containing all the elements that are common to both sets A and B.
** In the Venn Diagram, this is region II.

Example:
Given U = { S, A, N, A, N, T, O, N, I, O }
A = {S, A, N }
B = { A, N, T, O, N, I, O}
C = { }

VENN DIAGRAMS AND SET OPERATIONS

Find:

1. **A ∩ B**

   ANSWER: A ∩ B = { A, N }

2. **A ∩ U**

   ANSWER: A ∩ U = { S, A, N }

3. **A ∩ C**

   ANSWER: A ∩ C = { }

U = { S, A, N, A, N, T, O, N, I, O }
A = {S, A, N }
B = { A, N, T, O, N, I, O}
C = { }

Union

The union of sets A and B, symbolized by A ∪ B, is the set containing all the elements that are members of set A or of set B (or of both sets).

** In the Venn Diagram, this is regions I, II, and III.

Given
U = { a, b, c, d, e, f, g, h, I, j, k, l }
A = { a, e, l }
B = { b, c, d, e, f}
C = { d, e, f, g, h, I, j}
D = { }

Example:
Find:

1. **A ∪ B**

   ANSWER: A ∪ B = { a, b, c, d, e, f, l }

2. **B ∪ C**

   ANSWER: B ∪ C = { b, c, d, e, f, g, h, I, j }

3. B ∪ D

   ANSWER: B ∪ D = { b, c, d, e, f }

Venn Diagrams and Set Operations

The Meaning of AND and OR

The word or is generally used to mean union.
The word and is generally used to mean intersection.

The Relationship between $n(A \cup B)$, $n(A)$, $n(B)$, and $n(A \cap B)$

For any finite sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example: page 58 #84 (*Survey of Mathematics*)
The results of a survey of visitors in Hollywood, CA, showed that 27 visited the Hollywood Bowl, 38 visited Disneyland, and 16 visited both the Hollywood Bowl and Disneyland. How many people visited either the Hollywood Bowl or Disneyland?

Let A = those who visited the Hollywood Bowl
Then $n(A) = 27$

Let B = those who visited Disneyland
Then $n(B) = 38$

$A \cap B$ = those who visited both the Hollywood Bowl and Disneyland

So $n(A \cap B) = 16$

Thus, $A \cup B$ = the number of people who visited either the Hollywood Bowl *OR* Disneyland.

So $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 27 + 38 - 16$
$= 49$