

## Subsets

Set  $A$  is a subset of set  $B$ , symbolized by  $A \subseteq B$ , if and only if all the elements of set  $A$  are also elements of set  $B$ .

The symbol  $A \subseteq B$  indicates that "set  $A$  is a subset of  $B$ ."

Thus,  $A \not\subseteq B$  indicates that set  $A$  is not a subset of set  $B$ . To show this, we must find at least one element in set  $A$  that is not an element of set  $B$ .

Examples: Determine whether set  $A$  is a subset of set  $B$ .

1.  $A = \{\text{apple, orange, plum}\}$  and  $B = \{\text{plum, orange, apple}\}$

Yes

2.  $A = \{2, 3\}$  and  $B = \{1, 2, 3, 4, 5\}$

Yes

3.  $A = \{b, a, t, s\}$  and  $B = \{b, a, t\}$

NO

4.  $A = \{x \mid x \in \mathbb{N}\}$  and  $B = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is even}\}$

NO

5.  $A = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is even}\}$  and  $B = \{x \mid x \in \mathbb{N}\}$

Yes

Set A is a proper subset of set B, symbolized by  $A \subset B$ , if and only if all the elements of set A are elements of set B and set  $A \neq B$  (that is, set B must contain at least one element not in set A).

Example: True or False

1.  $\{ \text{red} \} \subset \{ \text{red}, \text{green}, \text{blue} \}$

True

2.  $\text{red} \subset \{ \text{red}, \text{green}, \text{blue} \}$

False,  $\text{red} \in \{ \text{red}, \text{green}, \text{blue} \}$

3.  $\{ \} \in \{ 1, 2, 3, 4 \}$

False,  $\{ \} \subset \{ 1, 2, 3, 4 \}$

4.  $\{ 1, 13, 27 \} \subseteq \{ 13, 27, 1 \}$

True.

But, is  $\{ 1, 13, 27 \} \subset \{ 13, 27, 1 \}$  true? NO

5.  $\{ 1 \} \in \{ \{ 1 \}, \{ 2 \}, \{ 3 \} \}$

True.  $\{ 1 \} \subset \{ \{ 1 \}, \{ 2 \}, \{ 3 \} \}$  is also true.

The number of distinct subsets of a finite set A is  $2^n$ , where n is the number of elements in set A.

Example:

- a) Determine the number of distinct subsets for the set  
 $A = \{ M, A, T, H \}$

$$2^n = 2^4 \\ = 16$$

- b) List all the distinct subsets for the set  $A = \{ M, A, T, H \}$

{ }	{ M, A }	{ M, A, T }
{ M }	{ M, T }	{ M, A, H }
{ A }	{ M, H }	{ A, T, H }
{ T }	{ A, T }	{ M, T, H }
{ H }	{ A, H }	{ M, A, T, H }
	{ T, H }	

- c) How many distinct subsets are proper subsets?

All but one