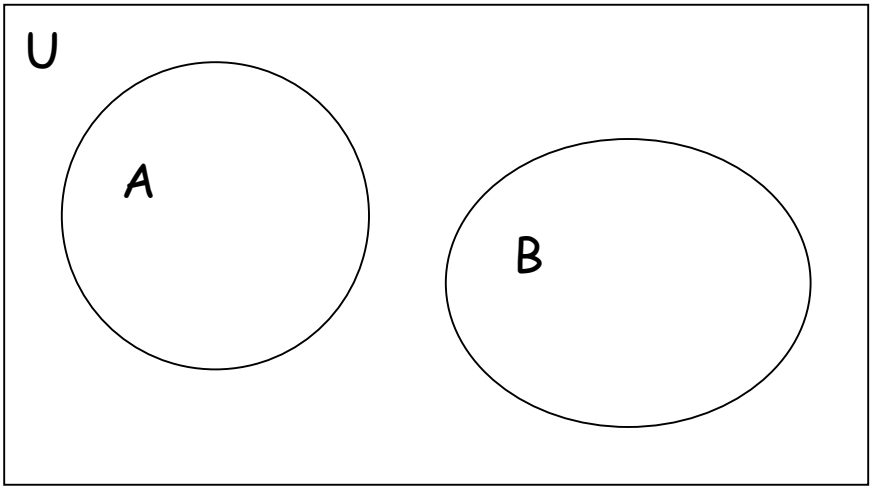
Venn Diagrams and Set Operations

Let U = the universal set (the set of all possible elements).
Two sets may be represented in a Venn Diagram in any of four different ways.

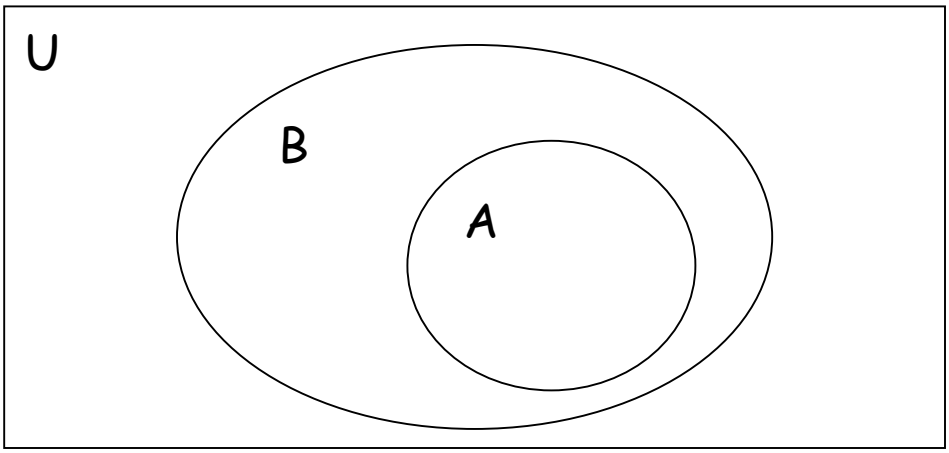
Case 1: Disjoint Sets

Two sets A and B are disjoint when they have no elements in common.

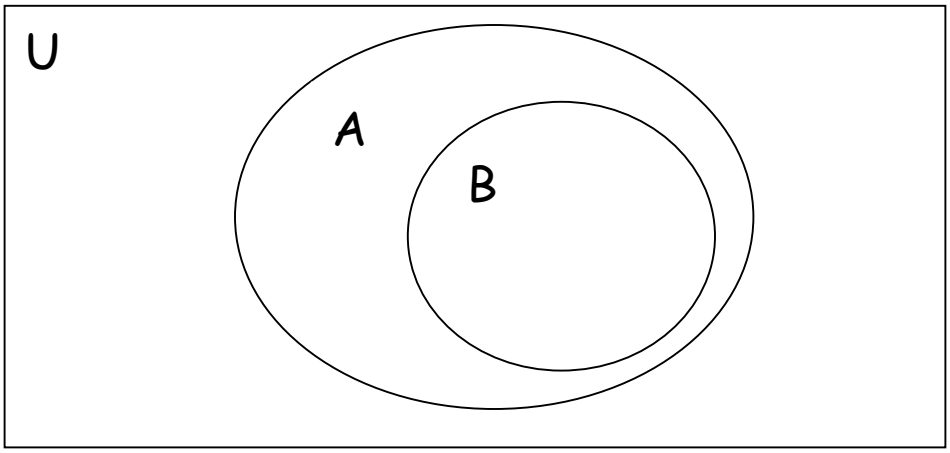


Case 2: Subsets

When $A \subseteq B$, every element of set A is also an element of set B. Thus, there can be no elements in set A that are not in set B.

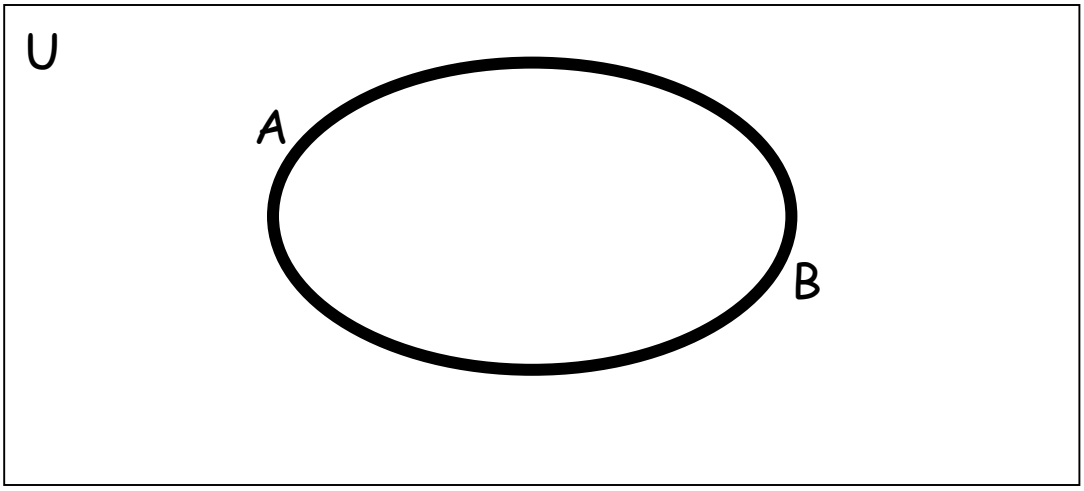


When $B \subseteq A$, every element of set B is also an element of set A. Thus, there can be no elements in set B that are not in set A.



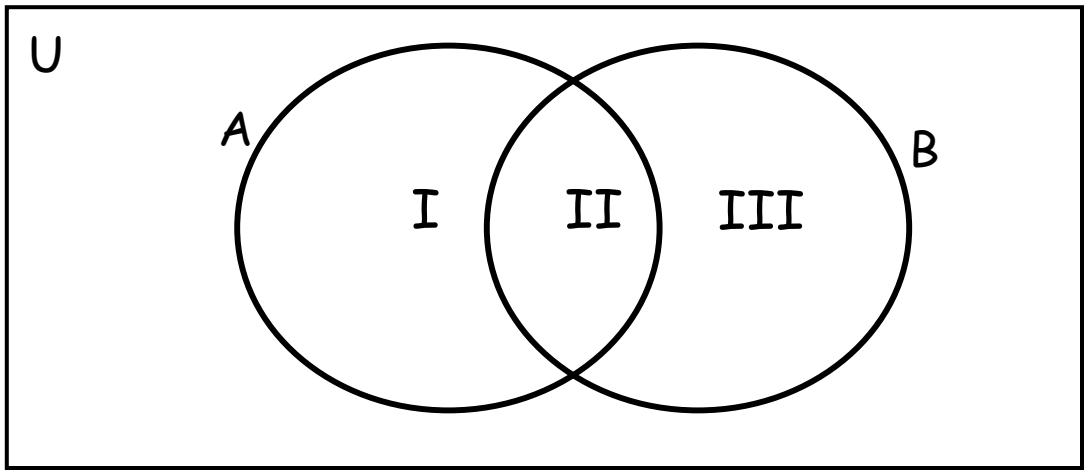
Case 3: Equal Sets

When set A = set B, all the elements of set A are elements of set B, and all the elements of set B are elements of set A.



Case 4: Overlapping Sets

When sets A and B have elements in common, those elements belong to region II. The elements that belong to set A but do not belong to set B are in region I. The elements that belong to set B but do not belong to set A are in region III.



Venn Diagrams will be helpful in understanding set operations.

Compliment

The compliment of set A, symbolized by A', is the set of all the elements in the universal set that are not in set A.

Example: Given U = { M, A, T, H, 1, 2, 0 } and A = { M, A, T, H }

Then A' = { 1, 2, 0 }

Intersection

The intersection of sets A and B, symbolized by $A \cap B$, is the set containing all the elements that are common to both sets A and B.

** In the Venn Diagram, this is region II.

Example:

Given U = { S, A, N, A, N, T, O, N, I, O }
A = {S, A, N }
B = { A, N, T, O, N, I, O}
C = { }

Find:

1. $A \cap B$

   $A \cap B = \{ A, N \}$

2. $A \cap U$

   $A \cap U = \{ S, A, N \}$

3. $A \cap C$

   $A \cap C = \{ \}$

Union

The union of sets A and B, symbolized by $A \cup B$, is the set containing all the elements that are members of set A or of set B (or of both sets).

** In the Venn Diagram, this is regions I, II, and III.

Example:

Given U = { a, b, c, d, e, f, g, h, I, j, k, l }
A = { a, e, I }
B = { b, c, d, e, f}
C = { d, e, f, g, h, I, j}
D = { }

Find:

1. $A \cup B$

   $A \cup B = \{ a, b, c, d, e, f, I \}$

2. $B \cup C$

   $B \cup C = \{ b, c, d, e, f, g, h, I, j \}$

3. $B \cup D$

   $B \cup D = \{ b, c, d, e, f \}$

The Meaning of AND and OR

The word or is generally used to mean union.
The word and is generally used to mean intersection.

The Relationship between $n(A \cup B)$, $n(A)$, $n(B)$, and $n(A \cap B)$

For any finite sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example: page 58 #84 (*Survey of Mathematics*)
The results of a survey of visitors in Hollywood, CA, showed that 27 visited the Hollywood Bowl, 38 visited Disneyland, and 16 visited both the Hollywood Bowl and Disneyland. How many people visited either the Hollywood Bowl or Disneyland?

Let A = those who visited the Hollywood Bowl
Then n(A) = 27

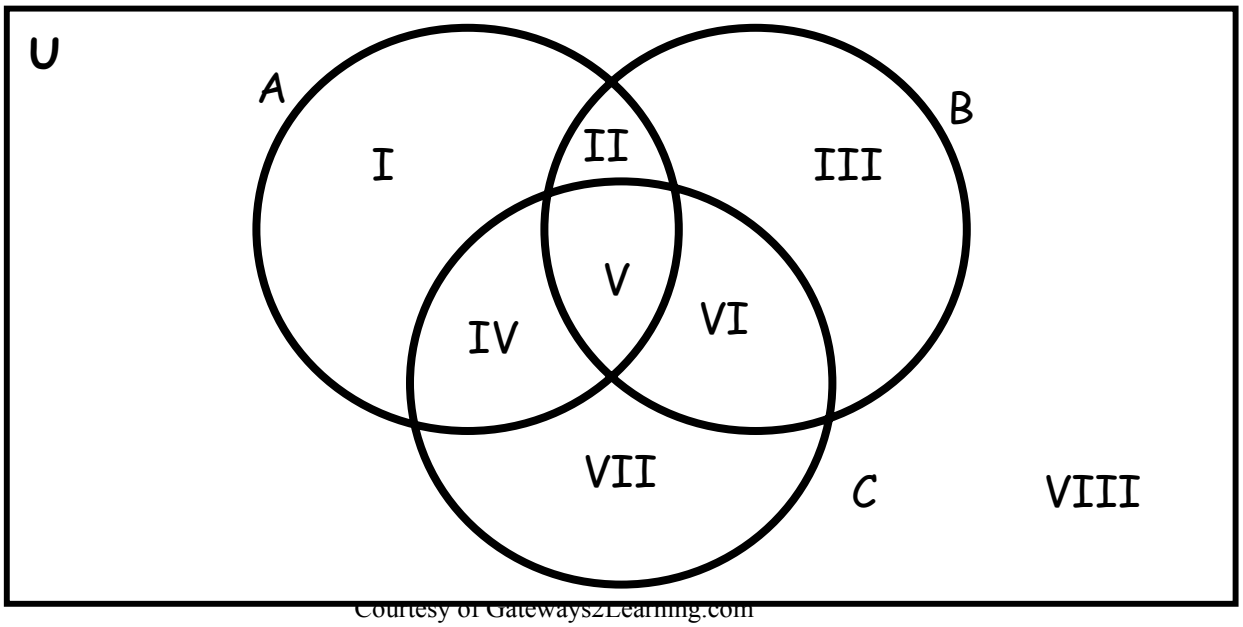
Let B = those who visited Disneyland
Then n(B) = 38

$A \cap B$ = those who visited both the Hollywood Bowl and Disneyland
So $n(A \cap B) = 16$

Thus, $A \cup B$ = the number of people who visited either the Hollywood Bowl *OR* Disneyland.

So $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 27 + 38 - 16$
$= 49$

Venn Diagrams with Three Sets



General Procedure for constructing Venn Diagrams with Three Sets A, B, and C

1. Determine the elements that are common to all three sets $(A \cap B \cap C)$. These will be placed in region V.

2. Determine the elements that are common to sets A and B only $(A \cap B \cap C')$. These elements are to be placed in region II.

3. Determine the elements that are common to sets A and C only $(A \cap C \cap B')$. These elements are to be placed in region IV.

4. Determine the elements that are common to sets B and C only $(B \cap C \cap A')$. These elements are to be placed in region VI.

5. Determine the elements that are found only in set A. These elements will be placed in region I.

6. Determine the elements that are found only in set B. These elements will be placed in region III.

7. Determine the elements that are found only in set C. These elements will be placed in region VII.

8. Finally, determine the elements that are found in the universal set but are not found in sets A, B, or C. These are to be placed in region VIII.

Example: page 65 #39 – 44 (*Survey of Mathematics*)
During a session of the U.S. Senate, three bills were voted on. The votes of six senators are shown in the table below. Determine in which region each senator would be placed. The set labeled bill 1 represents the set of senators who voted yes on bill 1, and so on.

Senator	Bill 1	Bill 2	Bill 3	Region
Grump	Yes	No	No	I
Happi	No	No	Yes	VII
Turwilliger	No	No	No	VIII
Dillinger	Yes	Yes	Yes	V
Isiatere	No	Yes	Yes	VI
Smith	No	Yes	No	III
Hutchinson	Yes	Yes	No	II
Graham	Yes	No	Yes	IV