

## Truth Tables

A [truth table](#) is a device used to determine when a compound statement is [true](#) or [false](#).

### Truth Tables for Negation, Conjunction, and Disjunction:

Negation:

$.p$	$\sim p$
T	F
F	T

Conjunction:

$.p$	$.q$	$.p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

<b>.p</b>	<b>.q</b>	<b>.p ∨ q</b>
T	T	T
T	F	T
F	T	T
F	F	F

Note:

The conjunction  $p \wedge q$  is true only when both p and q are true.

The disjunction  $p \vee q$  is false only when both p and q are false.

Examples:

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#10  $\sim(p \vee \sim q)$

<b>.p</b>	<b>.q</b>	<b><math>\sim q</math></b>	<b><math>p \vee \sim q</math></b>	<b><math>\sim(p \vee \sim q)</math></b>
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

#14  $(p \vee \sim q) \wedge r$

<b>.p</b>	<b>.q</b>	<b>.r</b>	<b><math>\sim q</math></b>	<b><math>p \vee \sim q</math></b>	<b><math>(p \vee \sim q) \wedge r</math></b>
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

Conditional:

Let  $p$  = You are a hard worker.

$q$  = You will get the promotion.

<b>.p</b>	<b>.q</b>	<b><math>p \rightarrow q</math></b>
T	T	T
T	F	F
F	T	T
F	F	T

← Broken Promise

The conditional statement  $p \rightarrow q$  is true in every case except when  $p$  is true and  $q$  is false.

Biconditional:

$p \leftrightarrow q$  means  $p \rightarrow q$  and  $q \rightarrow p$  or symbolically:  $(p \rightarrow q) \wedge (q \rightarrow p)$

$\cdot p$	$\cdot q$	$(p \rightarrow q)$	$\wedge$	$(q \rightarrow p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T

So in summary:

$\cdot p$	$\cdot q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Note:

The biconditional statement,  $p \leftrightarrow q$ , is true only when both  $p$  and  $q$  have the same truth value.

(i.e. both are True or both are False)

Examples:

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#14  $(\sim q \wedge p) \rightarrow \sim q$

$\cdot p$	$\cdot q$	$(\sim q$	$\wedge$	$p)$	$\rightarrow$	$\sim q$
T	T	F	F	T	T	F
T	F	T	T	T	T	T
F	T	F	F	F	T	F
F	F	T	F	F	T	T

This is what we call a tautology.



Answer column

#24  $[ r \wedge (q \vee \sim p)] \leftrightarrow \sim p$

.p	.q	.r	[ r	∧	( q	∨	~p )]	↔	~p
T	T	T	T	T	T	T	F	F	F
T	T	F	F	F	T	T	F	T	F
T	F	T	T	F	F	F	F	T	F
T	F	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T	T	T
F	T	F	F	F	T	T	T	F	T
F	F	T	T	T	F	T	T	T	T
F	F	F	F	F	F	T	T	F	T



Answer column

Step 1 in red.

Step 2 in purple.

Step 3 in green.

Step 4 in blue.