The Algebra of Functions

I. The Sum or Difference of Two Functions

- If f and g are functions and x is in the domain of both functions, then:
  - $(f+g)(x)=f(x)+g(x)$
  - $(f-g)(x)=f(x)-g(x)$

Example: For $f(x)=4x^2+2x-3$ and $g(x)=6x^2+5x-12$ find:

1. $(f+g)(x)$

$(f+g)(x)=f(x)+g(x)$ — Recall, the definition for adding functions.

$(f+g)(x)=(4x^2+2x-3)+(6x^2+5x-12)$ — Substitute the functions into the formula.

$(f+g)(x)=(4x^2+2x-3)+(6x^2+5x-12)$ — Remove parentheses and combine like terms.

$=4x^2+2x-3+6x^2+5x-12$

$=10x^2+7x-15$

Thus, $(f+g)(x)=10x^2+7x-15$

2. $(f-g)(x)$

$(f-g)(x)=f(x)-g(x)$ — Recall, the definition for subtracting functions.

$(f+g)(x)=(4x^2+2x-3)-(6x^2+5x-12)$ — Substitute the functions into the formula.

$(f+g)(x)=(4x^2+2x-3)-(6x^2+5x-12)$ — Remove parentheses and combine like terms. But don't forget to distribute the minus sign (or imaginary -1) through the second set of parentheses.

$=4x^2+2x-3-6x^2-5x+12$

$=4x^2+2x-3-6x^2-5x+12$

$=-2x^2-3x+9$

Thus, $(f-g)(x)=-2x^2-3x+9$

II. The Product or Quotient of Two Functions

- If f and g are functions and x is in the domain of both functions, then:
  - $(f\cdot g)(x)=f(x)\cdot g(x)$
  - $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

Examples: For $f(x)=-3x+1$ and $g(x)=x^2+2$, find the following:

1. $\left(\frac{f}{g}\right)(3)$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$$

Recall, the definition for the quotient of functions.

$$\left(\frac{f}{g}\right)(x)=\frac{-3x+1}{x^2+2}$$

Substitute the functions into the formula.

$$\left(\frac{f}{g}\right)(x)=\frac{-3(3)+1}{(3)^2+2}$$

Now, substitute the value for x into the quotient and simplify.

$$=\frac{-9+1}{9+2}$$

$$=-\frac{8}{11}$$

Thus, $\left(\frac{f}{g}\right)(3)=\boxed{-\frac{8}{11}}$.

2. $(f\bullet g)(4)$

$$(f\bullet g)(x)=f(x)\bullet g(x)$$

Recall, the definition for the product of functions.

$$f(4)=-3(4)+1$$

First, find $f(4)$

$$=-12+1$$

$$=-11$$

$$g(4)=(4)^2+2$$

Next, find $g(4)$

$$=16+2$$

$$=18$$

$$(f\bullet g)(4)=f(4)\bullet g(4)$$

Now plug these values in to the formula for the product.

$$=(-11)(18)$$

$$=-198$$

Thus, $(f\bullet g)(4)=\boxed{-198}$

III. <u>Determining the New Domain</u>

- The domain of $f+g$, $f-g$, or $f\bullet g$ is the set of all values common to the domains of f and g.
- The domain of $\frac{f}{g}$ is the set of all values common to the domains of f and g, excluding any values for which $g(x)=0$.