Solving Equations

I. Recall the Order of Operations

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

When simplifying expressions, we work from the top down (P E M D A S).
However, when solving equations, we will work to "undo" operations from the bottom up (S A D M E P). Of course, before undoing any operations, it is a good idea to simplify your equation as much as possible.

II. Combining Like Terms

A term is a number, a variable, or a product or quotient of variables and numbers. Like/similar terms are those terms whose variable components look exactly the same.

Examples: Simplify (i.e. combine like terms)

1. pg. 28 #32 in Intermediate Algebra by Bittinger

$$9a - 5a^2 + 4a$$

$\boxed{9a} - 5a^2 + \boxed{4a}$ The terms shown in red are the "like" terms. Thus, we can combine these terms.

$(9a + 4a) - 5a^2$ We combine the terms by first grouping them together. Notice that both terms in the parentheses have a common factor (a). Thus, we can factor out this term.

$(9 + 4)a - 5a^2$ Now, simplify the expression in the parentheses.

$$\boxed{13a - 5a^2}$$

Notice, there remaining terms do not have similar variables, so we can not simplify any further.

2. pg. 28 #38 in Intermediate Algebra by Bittinger

$$-9n + 8n^2 + n^3 - 2n^2 - 3n + 4n^3$$

$$-9n + 8n^2 + n^3 - 2n^2 - 3n + 4n^3$$

Similar or like terms are shown in the same color. (Also, notice that the sign in front of the term is the same color as it is.) We can group these terms together in order to combine them.

$$(-9n - 3n) + (8n^2 - 2n^2) + (n^3 + 4n^3)$$

We combine the terms by first grouping them together. Now, simplify the expressions in the parentheses by factoring.

$$(-9 - 3)n + (8 - 2)n^2 + (1 + 4)n^3$$

Now, simplify the expressions in the parentheses by adding/subtracting.

$$\boxed{-12n + 6n^2 + 5n^3}$$

There are no more "like" terms. Thus, the expression has been simplified.

3. pg. 28 #54 in Intermediate Algebra by Bittinger

$$7b - \{6[4(3b - 7) - (9b + 10)] + 11\}$$

For problems with several levels of parentheses, work from the inner most set outwards, simplifying as you go along.

$$7b - \{6[4(3b - 7) - (9b + 10)] + 11\}$$

First, distribute the negative/minus through this set of parentheses. (Shown in red)

$$7b - \{6[4(3b - 7) - 9b - 10] + 11\}$$

Next, distribute the 4 through this set of parentheses. (Shown in green)

$$7b - \{6[12b - 7 - 9b - 10] + 11\}$$

Now, simplify the expressions in the parentheses by adding/subtracting.

$$7b - \{6[12b - 7 - 9b - 10] + 11\}$$

$$7b-\{6[3b-17]+11\}$$

Next, distribute the 6 through the square brackets.

$$7b-\{18b-102+11\}$$
$$7b-\{18b-91\}$$

Now simplify the expression in the squiggly brackets by combining like terms.

$$7b-\{18b-91\}$$
$$7b-18b+91$$

Now, distribute the negative sign through the squiggly brackets.

$$7b-18b+91$$
$$\boxed{-11b+91}$$

Simplify by combining like terms.

III. Solutions to Linear Equations

Remember, there is always more than one way to solve an equation!

<u>Examples</u>: Solve. Be sure to check your answer.

1. pg. 28 #66 in <u>Intermediate Algebra</u> by Bittinger

$$210(x-3)=840$$
$$210x-630=840$$

First, we distribute the 210 through the parentheses.

$$210x-630=840$$
$$210x-630+630=840+630$$
$$210x-630+630=840+630$$
$$210x=1470$$

Next, we work to get the variable alone on one side of the equation by "undoing" the operations. (Reversing the order of operations) Thus, we add 630 to both sides of the equation and simplify.
** Remember, whatever you do to one side of the equation, you MUST also do it to the other side in order for the equation to stay balanced.

$$\frac{210x}{210}=\frac{1470}{210}$$
$$\boxed{x=7}$$

Now, isolate the x by dividing both sides by 210.

Now, we must CHECK our answers. We do this by plugging the value we found for x back in to the original equation and checking to make sure the equation remains equal.

$$210(x-3)=840$$
$$210(7-3)\overset{?}{=}840$$
$$210(4)\overset{?}{=}840$$
$$840\overset{!}{=}840$$

2. pg. 29 #78 in Intermediate Algebra by Bittinger

$$\frac{1}{2}(6m+48)-20=-\frac{1}{4}(12m-72)$$

First, we distribute the $\frac{1}{2}$ through the parentheses on the left and the $-\frac{1}{4}$ through the parentheses on the right.

$$\frac{1}{2}(6m+48)-20=-\frac{1}{4}(12m-72)$$
$$\frac{1}{2}(6m)+\frac{1}{2}(48)-20=-\frac{1}{4}(12m)+\left(-\frac{1}{4}\right)(-72)$$
$$3m+24-20=-3m+18$$

Next, simplify the left side of the equation by combining like terms.

$$3m+24-20=-3m+18$$
$$3m+4=-3m+18$$

Now, isolate the m by moving all of the variable terms to one side of the equals and the constant terms to the other side.

$$3m+4=-3m+18$$
$$3m+4-4=-3m+18-4$$
$$3m=-3m+14$$
$$3m+3m=-3m+3m+14$$
$$6m=14$$

Now, solve for the variable by dividing both sides by 6 and simplifying.

$$\frac{6m}{6}=\frac{14}{6}$$
$$\boxed{m=\frac{7}{3}}$$

Now, we must CHECK our answers. We do this by plugging the value we found for m back in to the original equation and checking to make sure the equation remains equal.

$$\frac{1}{2}(6m+48)-20=-\frac{1}{4}(12m-72)$$

$$\frac{1}{2}\left(6\left(\frac{7}{3}\right)+48\right)-20\overset{?}{=}-\frac{1}{4}\left(12\left(\frac{7}{3}\right)-72\right)$$

$$\frac{1}{2}(14+48)-20\overset{?}{=}-\frac{1}{4}(28-72)$$

$$\frac{1}{2}(62)-20\overset{?}{=}-\frac{1}{4}(-44)$$

$$31-20\overset{?}{=}11$$

$$11\overset{!}{=}11$$

<u>Note</u>: If we had made an error, the last step might look like the following:

$$13\neq 9$$

The two sides would not be equal. At that point, we would know that we needed to go back to our work and look for a mistake (or mistakes). If you can't find one, put the problem aside and try doing the problem completely over on another piece of paper WiTHOUT referring back to your original work, and see if you arrive at a different answer.