Other Equations of Lines

I. **Point-Slope Equations**

If we are given the slope of a line, m, and a point on the line (x_1, y_1) OR two points on a line (x_1, y_1) and (x_2, y_2), we can determine the equation of the line by using the point-slope equation of a line.

$$y - y_1 = m(x - x_1)$$

Examples: Determine the equation of a line given the following information.

1. $m = -\frac{2}{3}$ and passes thru the point $(6,5)$

$y - y_1 = m(x - x_1)$ — Recall the point-slope formula.

$y - 5 = -\frac{2}{3}(x - 6)$ — Substitute the given information in to the formula: $m = -\frac{2}{3}$, $x_1 = 6$ and $y_1 = 5$

Now simplify the equation until it is in the slope-intercept form.

$y - 5 = -\frac{2}{3}x + \left(-\frac{2}{3}\right)(-6)$ — First, we distribute the fraction through the parentheses.

$y - 5 = -\frac{2}{3}x + 4$

$y - 5 + 5 = -\frac{2}{3}x + 4 + 5$ — Next, isolate the y variable by adding 5 to both sides of the equation. Then, simplify.

$\boxed{y = -\frac{2}{3}x + 9}$ — This is the equation of our line in slope-intercept form.

2. The line passes thru $(-4,-7)$ and $(-2,-1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Recall the formula for slope.

$m = \frac{-7-(-1)}{-4-(-2)} = \frac{-7+1}{-4+2} = \frac{-6}{-2} = \boxed{3}$ — Use the formula to determine the slope of the line passing thru the two points.

$y - y_1 = m(x - x_1)$ — Now, recall the point-slope formula.

$y - (-7) = 3(x - (-4))$ — Substitute the given information in to the formula. You can choose to use either point. I will use $(-4,-7)$.

Now simplify the equation until it is in the slope-intercept form.

$$y+7=3(x+4)$$
First, we simplify the double negatives.

$$y+7=3x+12$$
Then, we distribute the slope through the parentheses.

Next, isolate the y variable by subtracting 7 to both sides of the equation.
$$y+7-7=3x+12-7$$

Then, simplify.

$$\boxed{y=3x+5}$$
This is the equation of our line in slope-intercept form.

II. Parallel and Perpendicular Lines

- Two lines are parallel if they have the same slope.
- Two lines are perpendicular if the product of their slopes is -1.
  - If one line has slope m, then the line that is perpendicular to it will have slope $-\frac{1}{m}$.
  - In otherwords, we take the reciprocal and change the sign.

Examples:

1. Find the equation of the line that is parallel to $2y=8x-2$ and passes thru the point $(-3,7)$.

$$2y=8x-2$$
$$\frac{2y}{2}=\frac{8x}{2}-\frac{2}{2}$$
$$y=4x-1$$
Put the equation of the line in point-slope form so that we may determine its slope.
Thus, the slope of the line is $m=4$.

Parallel lines have the same slope. Thus, our new line will have slope $m=4$.

$$y-y_1=m(x-x_1)$$
Now, recall the point-slope formula.

$$y-7=4(x-(-3))$$
Substitute the given information in to the formula.

Now simplify the equation until it is in the slope-intercept form.

$$y-7=4(x+3)$$
First, we simplify the double negatives.

$$y-7=4x+12$$
Then, we distribute the slope through the parentheses.

$$y-7+7=4x+12+7$$
Next, isolate the y variable by adding 7 to both sides of the equation. Then, simplify.

$$\boxed{y=4x+19}$$
This is the equation of our parallel line in slope-intercept form.

2. Find the equation of the line that is perpendicular to $2y = 8x - 2$ and passes thru the point $(-8,3)$.

$2y = 8x - 2$

$\frac{2y}{2} = \frac{8x}{2} - \frac{2}{2}$ Put the equation of the line in point-slope form so that we may determine its slope.

$y = 4x - 1$ Thus, the slope of the line is $m = 4$.

Parallel lines have the same slope. Thus, our new line will have slope $m = 4$.

$y - y_1 = m(x - x_1)$ Now, recall the point-slope formula.

$y - 3 = 4(x - (-8))$ Substitute the given information in to the formula.

Now simplify the equation until it is in the slope-intercept form.

$y - 3 = 4(x + 8)$ First, we simplify the double negatives.

$y - 3 = 4x + 32$ Then, we distribute the slope through the parentheses.

$y - 3 + 3 = 4x + 32 + 3$ Next, isolate the y variable by adding 3 to both sides of the equation. Then, simplify.

$\boxed{y = 4x + 35}$ This is the equation of our perpendicular line in slope-intercept form.

III. Applications

Example: (Intermediate Algebra, 6e by Bittinger page 124 #44)
Pressure at sea depth. The pressure 100 feet beneath the ocean's surface is approximately 4 atm (atmospheres), whereas at a depth of 200 feet, the pressure is about 7 atm.

a) Find a linear function that expresses pressure as a function of depth.

U Let P = the pressure

- Pressure depends on depth. Therefore, this is our dependent variable. Recall, y is usually our dependent variable.

Let d = the depth

- Depth is independent of any other variable. Therefore, this is our independent variable. Recall, x is usually our independent variable.

Thus, points on our line will be in the form (d, P)

So from the problem we have the points (100, 4) and (200, 7).

P & S Find the slope of the line passing thru the two points.

$$m = \frac{p_2 - p_1}{d_2 - d_1} = \frac{4-7}{100-200} = \frac{-3}{-100} = \boxed{0.03}$$

Now, use the point-slope equation to find the linear function.

$$p - p_1 = m(d - d_1)$$

$$p - 4 = 0.03(d - 100)$$
$$p - 4 = 0.03d - (0.03)(100)$$
$$p - 4 = 0.03d - 3$$
$$p - 4 + 4 = 0.03d - 3 + 4$$
$$p = 0.03d + 1$$

State A linear function that expresses pressure as a function of depth is $p = 0.03d + 1$ or in function format: $P(d) = 0.03d + 1$

b) Use the function in part (a) to determine the pressure at a depth of 690 feet.

U Let d = depth = 690 feet
P $P(d) = 0.03d + 1$
S $P(690) = 0.03(690) + 1$
$= 20.7 + 1$
$P(690) = 21.7$ atm

State The pressure at a depth of 690 feet below the ocean's surface is 21.7 atm.