Operations and Properties of Real Numbers

$\mathbb{R}$ = the Real Numbers

I. Absolute Values

Absolute value, $|a|$, represents the distance that the number a is from zero on a number line.

Can you measure a negative distance with a ruler?

NO

Then an absolute value will NOT return a negative value.

(* UNLESS someone puts a negative sign on the OUTSIDE of it.)

Examples: Evaluate

1. $|-3| = 3$

2. $|3| = 3$

3.
$$\begin{aligned} -|-3| &= -(|-3|) \\ &= -(3) \\ &= \boxed{-3} \end{aligned}$$

4.
$$\begin{aligned} -|3| &= -(|3|) \\ &= -(3) \\ &= \boxed{-3} \end{aligned}$$

5. $|0| = \boxed{0}$

II. Inequalities

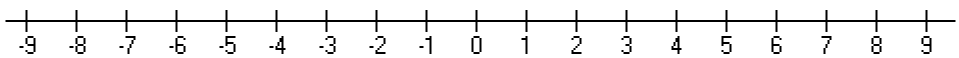
For any two numbers on a number line, the one farthest to the left is the lesser of the two numbers. The symbols $<$ and $>$ are used to compare numbers. The symbol $<$ means "is less than" ($\le$ means "is less than or equal to"), and the symbol $>$ means "is greater than" ($\ge$ means "is greater than or equal to"). These symbols are called inequalities.

__Example__: Write the meaning of each inequality and determine whether it is a true statement.
pg. 20 # 13 in __Intermediate Algebra__ by Bittinger

$-6 \le -2$

Meaning: -6 is less than or equal to –2

On a number line:



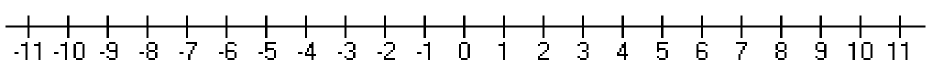
Notice that –6 is farther to the left on the number line than –2. Therefore, -6 is less than or equal to –2. Thus, the statement is TRUE.

__Example__: Write the meaning of each inequality and determine whether it is a true statement.

$-10 \ge 9$

Meaning: -10 is greater than or equal to 9

On a number line:



Notice that –10 is farther to the left on the number line than 9. Therefore, -10 is LESS than or equal to 9. Thus, the statement is FALSE.

III. Adding Real Numbers

If both numbers have the same sign, then add the absolute values and give the answer the same sign.

Example: Add

$$-23+(-56)=\boxed{-79}$$

If the numbers have **different signs**, then **subtract** the smaller absolute value from the larger absolute value and **give the answer** the **same sign as the number with the larger absolute value**.

(The above statement uses the correct mathematical terminology, but most students find it easier to remember the following statement:

If the numbers have **different signs**, then **subtract** and **give the answer** the **same sign as** the **"bigger" number**.

Once you understand a little more about positive and negative numbers, you will see why this statement is technically not the correct way to state the process, but I fully believe in using whatever terminology it takes to help YOU remember the correct process and arrive at the correct answer.)

Examples:

1. pg. 20 # 29 in Intermediate Algebra by Bittinger

$$-3.9+2.7=\boxed{-1.2}$$

2. pg. 20 # 36 in Intermediate Algebra by Bittinger

$$-\frac{1}{2}+\frac{4}{5} = -\frac{5}{10}+\frac{8}{10}$$

$$=\boxed{\frac{3}{10}}$$

3. pg. 20 # 42 in Intermediate Algebra by Bittinger

$$21.7+\left(-28.3\right)=\boxed{-6.6}$$

** For a more detailed explanation see Adding Integers page **

(found under the Basic Math heading)

4. $9.76+(-9.76)=\boxed{0}$

The real numbers above are what is known as additive inverses.

IV. Subtracting Real Numbers

Subtracting a real number is the same as adding the opposite of that real number.
In other words, change the subtraction problem to an addition problem and then follow the rules of addition.

** For a more detailed explanation see Subtracting Integers page **
(found under the Basic Math heading)

Examples: Subtract

1. pg. 20 # 62 in Intermediate Algebra by Bittinger

$$-3-(-9) \quad = -3+9$$
$$= \boxed{6}$$

i.e. The opposite of –9 is +9. OR Two negatives (i.e. the negative sign and the minus sign) make a positive (or a plus).

2. pg. 20 # 64 in Intermediate Algebra by Bittinger

$$-7-8 \quad = -7+(-8)$$
$$= \boxed{-15}$$

i.e. The opposite of 8 is –8.

V. Multiplying and Dividing Real Numbers

Remember the following when multiplying real numbers:

(Positive)(Positive) = POSITIVE
(Negative)(Positive) = NEGATIVE
(Positive)(Negative) = NEGATIVE
(Negative)(Negative) = POSITIVE

An alternative way to remember this is:
If the two numbers have the same sign, when you multiply, the answer will be positive.
If the two numbers have different signs, when you multiply, the answer will be negative.

Remember the following when dividing real numbers:

(Positive) / (Positive) = POSITIVE
(Negative) / (Positive) = NEGATIVE
(Positive) / (Negative) = NEGATIVE
(Negative) / (Negative) = POSITIVE

Recall that the fraction bar is another way of writing division.

An alternative way to remember this is:
If the two numbers have the same sign, when you divide, the answer will be positive.
If the two numbers have different signs, when you divide, the answer will be negative.

VI. The Sign of a Fraction

If a fraction is negative, the negative can be written in several locations:

On the top: $\frac{-a}{b}$

In front of the fraction: $-\frac{a}{b}$

On the bottom: $\frac{a}{-b}$

The only thing you can NOT do is list it on both the top and bottom at the same time!
(Why? Because that would be a negative divided by a negative which equals a positive.)

VII. The Law of Reciprocals

When two reciprocals are multiplied, their product is 1.

To find the reciprocal of a real number, make it into a fraction if it is not already one. Then, flip the fraction over. (Remember, you can change any whole number to a fraction by simply putting 1 in the denominator.)

Examples: Find the reciprocal of each real number.

$-97 \quad \Rightarrow$ first change to a fraction

$$\Rightarrow \frac{-97}{1}$$

$\Rightarrow$ now flip the fraction over

$$\Rightarrow \frac{1}{-97}$$

$$\Rightarrow \boxed{-\frac{1}{97}}$$

I do not like to leave negatives in the denominators of fractions. It is a bad practice. Negatives in the denominators have a way of getting lost. Don't do it! Move it out to the front or put it in the numerator.

VIII. Division by Zero

YOU CAN NEVER DIVIDE BY ZERO !!!!

The answer to a division by zero is said to be undefined.

IX. The Commutative Laws

For any real numbers a and b,

For addition: $a + b = b + a$

For multiplication: $ab = ba$

Note:

Subtraction and division are NOT commutative.

This law simply states that with addition and multiplication of numbers, you can change the order of the numbers in the problem and it will not affect the answer.

X. The Associative Laws

For any real numbers a, b and c,

For addition: $a + (b + c) = (a + b) + c$

For multiplication: $a(bc) = (ab)c$

Note:
Subtraction and division are NOT associative.

This law simply states that with addition and multiplication of numbers, you can change the grouping of the numbers in the problem and it will not affect the answer.

XI. The Distributive Law

For any real numbers a, b and c,

$$a(b+c) = ab + ac$$
$$(b+c)a = ba + ca$$

When dealing with the distributive law, just remember to multiply everything on the inside of the parentheses by the number on the outside of the parentheses.

Examples: Write an equivalent expression using the distributive law.

1. pg. 21 # 138 in Intermediate Algebra by Bittinger

$$5x(y-z+w) = 5x(y) + 5x(-z) + 5x(w)$$
$$= \boxed{5xy - 5xz + 5xw}$$

2.

$$-27m(3p-4r) = -27m(3p) + (-27m)(-4r)$$
$$= -81mp + 108mr$$

Be careful when there is a negative sign on the number on the outside of the parentheses. It will change the signs of the numbers on the inside of the parentheses

Examples: Find an equivalent expression by factoring.
(This simply means reverse the distributive property.)

1. pg. 21 # 142 in Intermediate Algebra by Bittinger

$$15x - 3$$

Look at both terms of the expression.
Both terms are divisible by 3. Thus, we can factor out the 3.

$$15x - 3 = 3(5x) + 3(-1)$$
$$= \boxed{3(5x - 1)}$$

2. $91x^2 - 65xy + 117x$

Look at both terms of the expression. All terms are divisible by $13x$. Thus, we can factor out the $13x$.

$$91x^2 - 65xy + 117x = 13x(7x) + 13x(-5y) + 13x(9)$$
$$= \boxed{13x(7x - 5y + 9)}$$