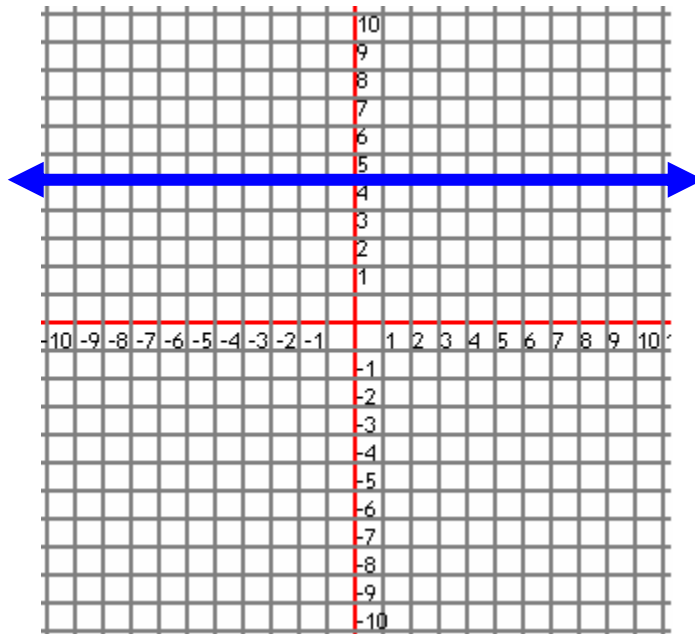


## Another Look at Linear Graphs

### I. Horizontal Lines

Every horizontal line has a slope of 0 (zero).

Below is the graph of the line  $y = 5$  (Shown in blue.)



To test this, choose any two points on the line.

i.e.  $(-6, 5)$  and  $(4, 5)$

Now use the slope formula to find the slope between these two points.

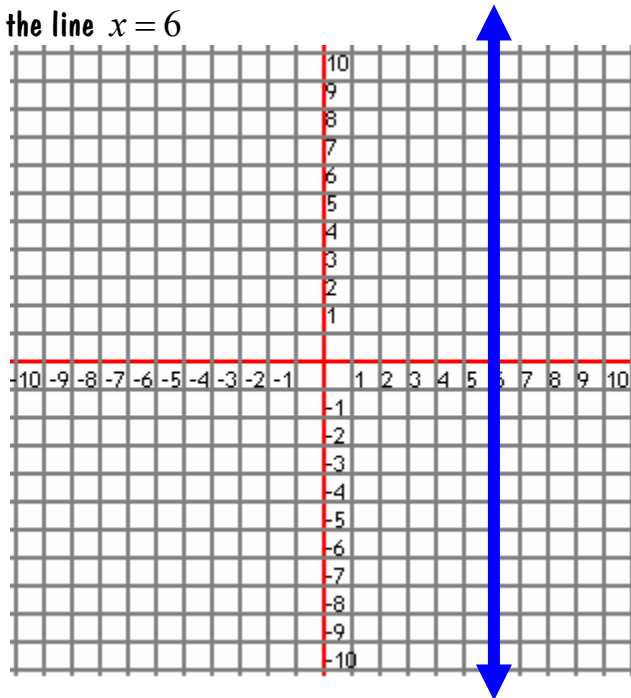
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{-6 - 4} = \frac{0}{-10} = \boxed{0}$$

The graph of any constant function of the form  $f(x) = b$  or  $y = b$  is a horizontal line that crosses the  $y$ -axis at  $(0, b)$  and has slope 0.

### II. Vertical Lines

The slope of any vertical line is **undefined**. We can also say the line has **NO** slope.

Below is the graph of the line  $x = 6$



To test this, choose any two points on the line.

i.e.  $(6, 5)$  and  $(6, -4)$

Now use the slope formula to find the slope between these two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{6 - 6} = \frac{9}{0}$$

Since you can not divide by zero, this value is undefined.

The graph of any constant equation of the form  $x = a$  or  $y = b$  is a vertical line that crosses the x-axis at  $(a, 0)$  and has no slope.

### III. Graphing Lines Using the x- and y-Intercepts

To determine the intercepts:

- The x-intercept is  $(a, 0)$ . To find  $a$ , let  $y = 0$  and solve the original equation for  $x$ .
- The y-intercept is  $(0, b)$ . To find  $b$ , let  $x = 0$  and solve the original equation for  $y$ .

**Examples:**

Find the intercepts. Graph the line using the intercepts, then choose a third point on the line and test it using the original equation.

(Intermediate Algebra, 6e by Bittinger; page 115 #36)

$$3x + 2y = 12$$

Solve for the x-intercept:

$$3x + 2(0) = 12$$

$$3x + 0 = 12$$

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

First, let  $y = 0$ .

Next, solve for  $x$ .

Thus, the x-intercept is  $(4, 0)$ .

Solve for the y-intercept:

$$3(0) + 2y = 12$$

$$0 + 2y = 12$$

$$2y = 12$$

$$\frac{2y}{2} = \frac{12}{2}$$

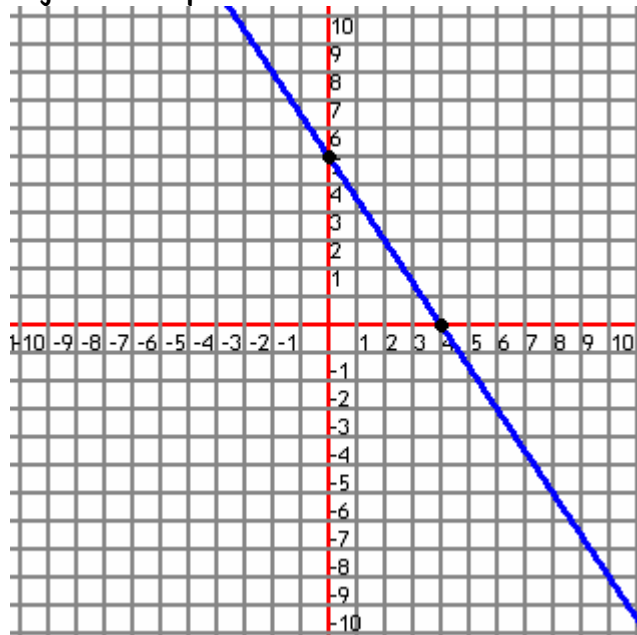
$$y = 6$$

First, let  $x = 0$ .

Next, solve for  $y$ .

Thus, the y-intercept is  $(0, 6)$ .

Next, graph the line using the intercepts:



Now, choose a third point on the line to test against the original equation.

I will choose  $(6, -3)$ . So, I will plug  $x = 6$  and  $y = -3$  in to the equation and test for equality.

$$\begin{aligned}3x + 2y &= 12 \\3(6) + 2(-3) &\stackrel{?}{=} 12 \\18 + (-6) &\stackrel{?}{=} 12 \\12 &\stackrel{!}{=} 12\end{aligned}$$

Thus, our line has been drawn correctly.

If when you choose a third point on the line and the equation does not yield equality, then you have not graphed your equation correctly.

#### IV. Forms of Linear Equations

There are several forms of linear equations:

- **Slope-Intercept Form:**
  - $f(x) = mx + b$
  - $y = mx + b$
- **Standard Form**
  - $Ax + By = C$  where  $A, B,$  and  $C$  are real numbers and  $A$  and  $B$  are not both zero.
- **Other Forms:**
  - $By = Ax + C$

#### V. Solving Equations Graphically

**Example:**

(Intermediate Algebra, 6e by Bittinger; page 116 #62)

**Copying Costs.** A local Mailboxes Etc store charges \$2.25 for binding plus 5 cents per page for each spiral bound copy of a town report. Estimate the length of a spiral bound report that costs \$3.50.

**Remember to use the Five Step Problem Solving Plan!**

First determine and graph an equation for the general situation:

U     Base fee = \$2.25 (this fee is constant and not changed by the number of pages.)

Thus, this will be the  $C$  value in our equation  $By = Ax + C$

Price per page = 5 cents = \$0.05

Let  $x$  = the number of pages

Let  $y$  = the total cost for binding and copying.

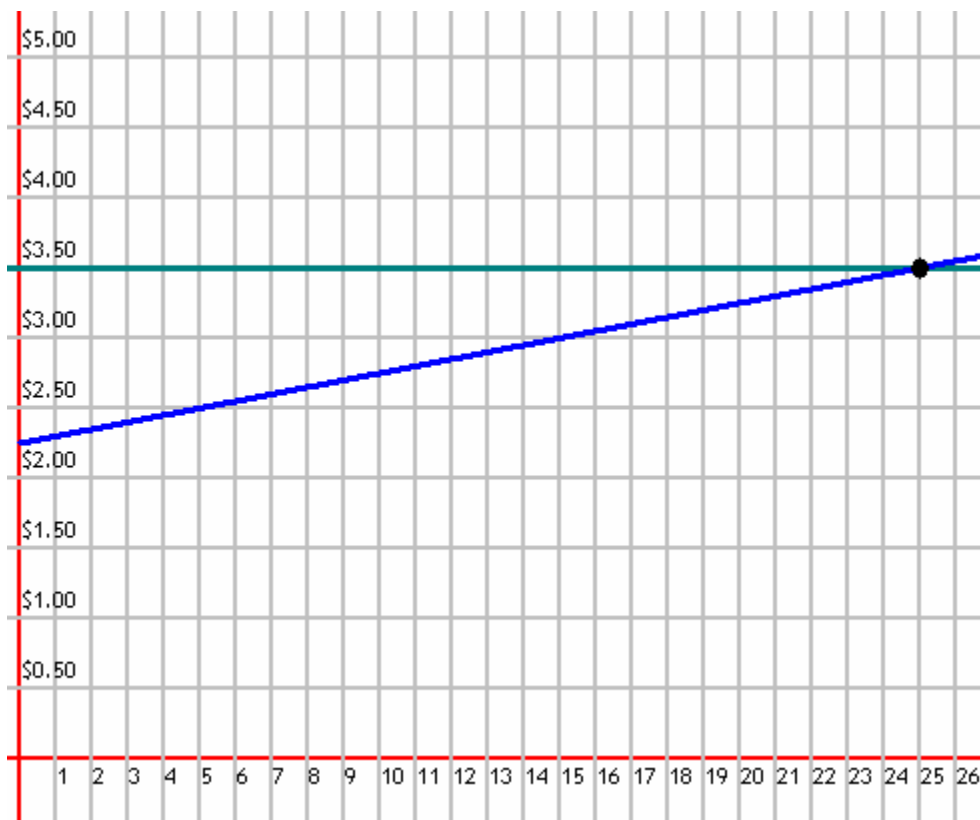
**P** So the equation that represents a general model for our problem is

$$y = 2.25 + 0.05x$$

**S** Now to solve this problem graphically, we will graph our general mathematical model

( $y = 2.25 + 0.05x$ ) and the specific model where the total cost is \$3.50

( $y = 3.50$ )



The green line represents the graph of  $y = 3.50$  and the blue line represents the graph of  $y = 2.25 + 0.05x$ . The solution to the problem is the point where the two lines intersect. In this problem, the lines intersect at  $(25, 3.50)$ . Recall, the  $x$  value represents the number of pages and the  $y$  value represents the total cost.

<b>State</b>	<b>Therefore, the length of a spiral bound report costing \$3.50 is 25 pages.</b>
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