Formulas, Models, and Geometry

I. Recall the Five Step Problem Solving Plan

U: Understand the problem
P: Make a plan. (translate into an algebraic equation)
S: Solve the problem
Check your answer for reasonableness!
State your answer in a complete sentence.

** You WILL loose points if you fail to do the last step. **

For a more detailed explanation of each of these steps, see the Five Step Plan for Word Problems handout. (can also be found on the web site under Basic Math pages)

II. Solving Formulas

A **formula** is an equation that uses letters to represent a relationship between two or more quantities.

Some commonly used formulas are:

$A = lw$; $F = ma$; $A = p\left(1 + \frac{r}{n}\right)^{nt}$; etc.

We can solve formulas for specified letters (or missing values).

Steps:

1. Get all terms containing the letter that is being solved for on one side of the equal sign and all other terms on the other side of the equal sign. (This one step may actually take several mathematical steps to complete in the problem.) To do this may require that you remove parentheses. To remove parentheses, either divide both sides by the multiplier in front of the parentheses or use the distributive law.

2. When all terms with the specified letter are on the same side, factor (if necessary) to combine like terms.

3. Solve for the letter in question by dividing both sides of the equation by the multiplier of that letter.

Examples: Solve.

1. pg. 44 #10 in Intermediate Algebra by Bittinger

$F = \frac{mv^2}{r}$, for m (a physics formula)

Steps

1 $F = \frac{mv^2}{r}$ — Step 1 is not necessary for this formula.

2 $F = m\left(\frac{v^2}{r}\right)$ — Factor out the m on the right side of the equation.

3 Divide both sides by $\left(\frac{v^2}{r}\right)$, the multiplier of m. Then, simplify.

$$F \div \left(\frac{v^2}{r}\right) = m\left(\frac{v^2}{r}\right) \div \left(\frac{v^2}{r}\right)$$

$$F\left(\frac{r}{v^2}\right) = m$$

2. pg. 44 #20 in Intermediate Algebra by Bittinger

$T = \frac{3}{10}(I - 12{,}000)$ for I (a tax formula)

Steps

1 $T = \frac{3}{10}(I - 12{,}000)$ — To remove parentheses, divide both sides of the equation by the multiplier on the outside of the parentheses. Then, simplify.

$$T \div \frac{3}{10} = \left(\frac{3}{10} \div \frac{3}{10}\right)(I - 12{,}000)$$

$$\frac{10}{3}T = I - 12{,}000$$

2 Add 12,000 to both sides to isolate the I. Then, simplify

$$\frac{10}{3}T + 12{,}000 = I - 12{,}000 + 12{,}000$$

$$\boxed{\frac{10}{3}T + 12{,}000 = I}$$

3 Not necessary in this problem.

III. Mathematical Models

A **mathematical model** can be a formula or a set of formulas developed to represent a real-world situation. In our problem solving plan, we form our mathematical models in the **P**lan step.

Examples: Solve.

1. pg. 45 #44 in Intermediate Algebra by Bittinger

Fencing. A rectangular garden is being constructed, and 76 feet of fencing is available. The width of the garden is to be 13 feet. What should the length be in order to use just 76 feet of fence?

U Fencing goes around the outside of the rectangle. Thus, 76 feet represents the perimeter of the garden (rectangle). So our formula is $P = 2l + 2w$, where $l = \text{length}$ and $w = \text{width}$. We know the width is to 13 feet. We want to find the length.

P
$$P = 2l + 2w$$
$$76 = 2l + 2(13)$$

S
$$76 = 2l + 26$$
$$76 - 26 = 2l + 26 - 26$$
$$50 = 2l$$
$$\frac{50}{2} = \frac{2l}{2}$$
$$\boxed{25 = l}$$

Check Is the answer reasonable? To determine this, check to see if the perimeter of a garden that is 25 feet by 13 feet is indeed 76 feet.

$$2(25) + 2(13) \stackrel{?}{=} 76$$
$$50 + 26 \stackrel{?}{=} 76$$

$$76 \overset{!}{=} 76$$

Thus, our answer is reasonable.

State The length of the garden should be 25 feet in order to use just 76 feet of fence.